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Investigation of Mistuning Effects on Cascade Flutter Using a Coupled Method

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Effects of frequency mistuning on cascade flutter are studied using a computational method with coupled structural dynamics and aerodynamics. An implicit finite-volume scheme of second-order accuracy is used to solve the unsteady Euler equations. The structural equations with bending and torsional degrees of freedom for a rigid blade profile are integrated in time simultaneously with the flow equations. Investigations are performed on flutter of a turbine cascade in bending motion and with alternate mistuning of the structural eigenfrequency. Previous work by an uncoupled method shows a beneficial effect of alternating frequency mistuning on flutter stability. This paper shows that fluid–structure interaction tends to decrease the effective amount of mistuning. There exists a minimum amount of mistuning required to stabilize the cascade. The same qualitative behavior is shown to exist with a compressor cascade.

Nomenclature

b	=	half-chord of blade
C_l	=	lift coefficient
C_m	=	moment coefficient about the elastic axis
h	=	translational displacement of the blade
K_h	=	bending stiffness of the blade
K_α	=	torsional stiffness
k_c	=	reduced frequency, $\omega_\alpha c/2U_\infty$
m	=	mass per unit span of blade
$r_\alpha b$	=	radius of moment of inertia about elastic axis
$x_\alpha b$	=	distance between center of gravity and elastic axis
α	=	rotational displacement of the blade
μ	=	mass ratio, $m/\pi\rho_\infty b^2$
ω_h	=	natural angular frequency for translation
ω_α	=	natural angular frequency for rotation

Introduction

IN flutter investigations a turbomachinery cascade is usually assumed to be tuned, which means that all blades in the row have identical structural and geometrical properties. However, due to manufacture tolerances, a real blade row exhibits variations in blade mass, geometry, and stiffness. A cascade in which not all blades are identical is said to be *mistuned*. The question arises as to how mistuning affects flutter stability and forced response. The first studies include those by Whitehead [1] and Ewins [2]. In many cases it was found that mistuning has a beneficial effect on flutter stability. On the other hand, mistuning may have an adverse effect on forced response. However, such adverse effect may be eliminated by carefully arranging the mistuning pattern.

In the case of flutter, alternate frequency mistuning, where the structural eigenfrequencies of neighboring blades alternate between a high and a low value, was found to have a stabilizing effect. Kaza and Kielb [3] performed a modal analysis of tuned and mistuned cascades of pretwisted fan blades. Linear theory for a flat plate was applied to solve for the flow ranging from subsonic to supersonic along the rotor span. An unshrouded fan stage, unstable at the design point, was shown to be stabilized by alternate mistuning, with a frequency variation of 7%.

Imregun and Ewins [4] performed numerical studies on a cascade of flat plates, in the incompressible, subsonic, and supersonic Mach number range. The structural behavior was modeled with a lumped parameter representation of rigid blade profiles, allowing for structural coupling between the blades. Alternate mistuning was found to stabilize critical vibration modes at the expense of damped ones.

An experimental investigation on forced response of naturally and artificially mistuned propfan blades was done by Mehmed and Murthy [5]. Intentional mistuning was performed by varying the ply orientation of the composite material of the blades, causing a variation of stiffness and vibration mode shape. This intentional mistuning was found to cause a large reduction in the forced response of higher responding blades with relatively little change in the amplitudes of lower responding blades, and therefore, alternate mistuning had an overall stabilizing effect.

An experimental and numerical investigation on an annular turbine cascade was done by Nowinski and Panovsky [6]. The blades were oscillated in a harmonic torsional mode. Three vibrational modes of the blades were tested: the traveling wave mode, the single blade mode, and the alternating blade mode. In the last test mode, only alternate blades in the cascade were excited in a traveling wave pattern while others remained stationary to simulate frequency mistuning. The authors found that alternate frequency mistuning reduced the dependence of the aerodynamic damping coefficient on the interblade phase angle (IBPA) and significantly enhanced the stability of the tested low-pressure turbine cascade.

In a previous work [7] by the present authors, mistuning was studied in the time domain by directly solving the unsteady flow of a cascade under mistuned oscillations. A multigrid time-accurate Navier–Stokes code is used to calculate quasi-three-dimensional unsteady flows around multiple oscillating turbine blades. The code is made parallel by using message passing interface (MPI) so that multiple passages can be calculated without the use of phase-shifted periodic boundary conditions. The uncoupled approach was used in which the oscillation frequencies and amplitudes of all blades are specified and stability is determined by calculating the aerodynamic damping coefficient as defined by Bölcs and Fransson [8]. The standard configuration 4 of a turbine cascade compiled by Bölcs and Fransson [8] was used as a test case. Damping coefficients were obtained for various interblade phase angles for the tuned case and compared with results for both phase-shift mistuning and frequency mistuning. It was found that mistuning of phase shift has small

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effects on the flutter characteristics. However, mistuning of frequency has the effect of averaging out the damping coefficient for the tuned blade row over the whole range of IBPA because of a temporally changing phase difference between each blade and its neighbors.

In summary, several studies have shown numerically that frequency mistuning has the effect of increasing the flutter stability by distributing the energy of unstable flutter modes over a range of stable modes. A mistuned cascade is not able to oscillate in a mode of constant IBPA. Therefore, at any time, several modes of different IBPAs are present, and as long as most of these modes are stable, the cascade will be overall stable. These findings have been confirmed by a perturbation analysis of Campobasso and Giles [9].

However, none of the previous investigations answers the question about the minimum amount of alternate mistuning that is required for stabilization. If the preceding description for the stabilizing mechanism of mistuning were complete, it would work at any finite amount of mistuning. It is obvious that this cannot be true, because we know that a real blade row is slightly mistuned but may still flutter. To investigate the effects of mistuning more accurately, a more adequate model of the physics has to be applied by using a coupled method that accounts for the interaction between fluid and structure.

The uncoupled method implies the assumption that the blade mass ratio is sufficiently high so that blades are oscillating at their structural eigenfrequencies and at a constant amplitude. It was suggested [7] that the definition of stability, based on the damping coefficient as defined in [8], may not be suitable in the presence of coupling between aerodynamics and structural dynamics. With frequency mistuning, the uncoupled method predicts a temporal maximum in the work done on each blade. This maximum exists even if the overall behavior is stable, because due to a time-dependent IBPA, the cascade will temporally go through an IBPA range in which the oscillation is unstable. If coupling between aerodynamics and structural dynamics is present, the work done on a blade will affect the blade's oscillation amplitude. Therefore, even in situations where the energy method predicts stability, the amplitude may temporally exceed a critical level. To prove this conjecture, flutter investigations have to be performed with a coupled approach.

To perform coupled computations, the present paper includes a structural model to account for fluid–structure interactions. An integrated method is used, which solves the Navier–Stokes and structural equations simultaneously in each real-time step. The elastic behavior of a blade is modeled with a linear spring for the bending motion and a torsional spring for the rotational degree of freedom. The method was used to investigate nonlinear flutter and results by the Navier–Stokes equations were compared with those by the Euler equations in [10,11]. In this work the effects of fluid–structure coupling on frequency mistuning are investigated. Because of fluid–structure interaction the actual oscillation frequency differs from the mistuned structural eigenfrequency. The effective mistuning is not given by the variation of the structural eigenfrequency, but by the variation of the actual oscillation frequency. It is found that fluid–structure coupling tends to diminish the differences between the oscillation frequencies of neighboring blades.

Flow Solver

For a two-dimensional control volume V with moving boundary ∂V the unsteady quasi-three-dimensional Favre-averaged Navier–Stokes equations with a k–ω turbulence model can be written as follows:

$$\frac{\partial}{\partial t}\iint_V \theta(x)\boldsymbol{w}\,\mathrm{d}V + \oint_{\partial V}\theta(x)(\boldsymbol{f}\,\mathrm{d}S_x + \boldsymbol{g}\,\mathrm{d}S_y) = \oint_{\partial V}\theta(x)(\boldsymbol{f}_\mu\,\mathrm{d}S_x + \boldsymbol{g}_\mu\,\mathrm{d}S_y) + \iint_V \boldsymbol{S}\,\mathrm{d}V \tag{1}$$

where the vector $\boldsymbol{w}$ contains the conservative flow variables plus the turbulent kinetic energy k and the specific dissipation rate ω, in the k–ω turbulence model by Wilcox [12]. The vectors $\boldsymbol{f}$, $\boldsymbol{f}_\mu$, $\boldsymbol{g}$, $\boldsymbol{g}_\mu$ are the Euler fluxes and viscous fluxes in the x- and y-directions, respectively. This formulation is quasi-three-dimensional in the sense that it accounts for a streamwise variation of the blade span by including the streamtube thickness $\theta(x)$. A detailed description of these terms can be found in [13].

The Jameson–Schmidt–Turkel (JST) scheme [14] is used for flux discretization with an implicit formulation, using a second-order difference for the time derivative. The equations are integrated in time by Jameson's pseudotime stepping [15]. The computation is performed in parallel by several processors, where each processor calculates the flow through one passage. The exchange of boundary conditions between neighboring blade passages is performed using the message passing interface.

Structural Model

The structural dynamic equations for a rigid blade profile with 2 degrees of freedom is written as follows:

$$[M]\frac{\mathrm{d}^2\boldsymbol{q}}{\mathrm{d}\tau^2} + [K]\boldsymbol{q} = \boldsymbol{F} \tag{2}$$

where

$$[M] = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix} \quad \text{and} \quad [K] = \begin{bmatrix} \left(\frac{\omega_h}{\omega_\alpha}\right)^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix}$$

are the nondimensional mass and stiffness matrices, and

$$\boldsymbol{F} = \frac{1}{\pi\mu k_c^2(\omega_\alpha/\omega_h)^2}\begin{Bmatrix} -C_l \\ 2C_m \end{Bmatrix}, \qquad \boldsymbol{q} = \begin{Bmatrix} \frac{h}{b} \\ \alpha \end{Bmatrix}$$

are the load and displacement vectors, respectively. The time τ is nondimensionalized by the eigenfrequency of the pitching mode, i. e., $\tau = \omega_\alpha t$.

The modal structural equations are discretized and cast into a pseudotime problem similar to the treatment of the flow equations. A Runge–Kutta scheme is used to simultaneously advance the structural and flow equations in pseudotime until convergence within each real-time step. Notice that the aeroelastic equations are implicitly coupled to the flow equations because the forcing term contains both C_l and C_m. By simultaneously iterating the flow and structural equations with the same pseudotime approach, this coupling is taken into account. Details of the method can be found in [10,11].

Mistuned blades are known to introduce structural mode split of the blade assembly. This effect is not considered in the present paper. The focus of the paper is on the effect of aerodynamic coupling on mistuned blades.

Results

Uncoupled Simulations

In [7], the effect of frequency mistuning was investigated by an uncoupled approach, where only the flow equations are solved with prescribed motion of the blades. With the uncoupled approach, the cascade behavior is periodic because of the specified sinusoidal blade motion. In that case, the work that the aerodynamic forces perform on a blade over one oscillation cycle is used as a measure for stability. If this work coefficient is positive, the cascade is unstable at the given IBPA.

Figure 1 shows the work coefficient as a function of the interblade phase angle for the standard configuration 4 by Bölcs and Fransson [8], which is a cascade of turbine blades oscillating in pure plunging mode in high subsonic flow. For pure bending the work coefficient is defined as

$$C_W = \frac{T_{\text{tuned}}}{T_{\text{total}}}\int_0^{T_{\text{total}}} C_h\,\mathrm{d}h \tag{3}$$

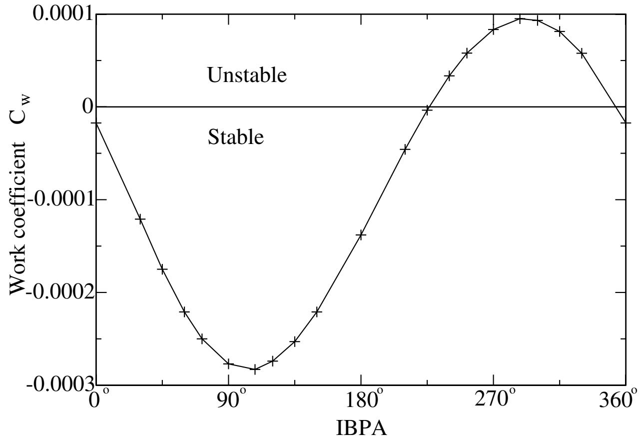

Fig. 1 Work coefficient of the tuned cascade vs interblade phase angle, uncoupled.

where h is the translational displacement divided by the chord length, C_W is the work coefficient, C_h the force coefficient in h-direction, T_{tuned} the period of the tuned case, and T_{total} a long overall period of the system determined by the amount of mistuning.

The focus of this paper is to investigate the coupled effect of aerodynamics and structures. In particular, we will demonstrate the phenomenon of frequency "lock-in" of the mistuned blades and the existence of minimum mistuning needed for flutter stability. For this purpose, only solutions of the Euler equations are presented. They suffice to elucidate the physical phenomena discovered. Quantitative predictions for real industrial configurations, however, should be made by performing Naiver–Stokes computations.

The calculation is performed on a number of blade passages, depending on the imposed interblade phase angle. These results were obtained by the uncoupled method. As shown in Fig. 1, the cascade exhibits flutter instability in a range of IBPAs between 240 and 360 deg.

However, when the cascade is subject to alternate mistuning, where the oscillation frequency of every second blade is increased over the nominal frequency by a given amount $\Delta\omega$, the uncoupled simulations in [7] yield negative average C_W defined by Eq. (3), i.e., the cascade is stabilized regardless of the magnitude of $\Delta\omega$. To examine the effect of $\Delta\omega$, we examine the transient work on the blade. Figure 2 shows the time history of work done on the cascade for mistuning levels of $\Delta\omega = 2, 5,$ and 10% of the nominal frequency ω. Here, time t is nondimensionalized by the nominal oscillation period $T = T_{\text{tuned}}$. The accumulated work at time t is defined as the sum of the nondimensional work performed on the cascade within the time interval between 0 and t.

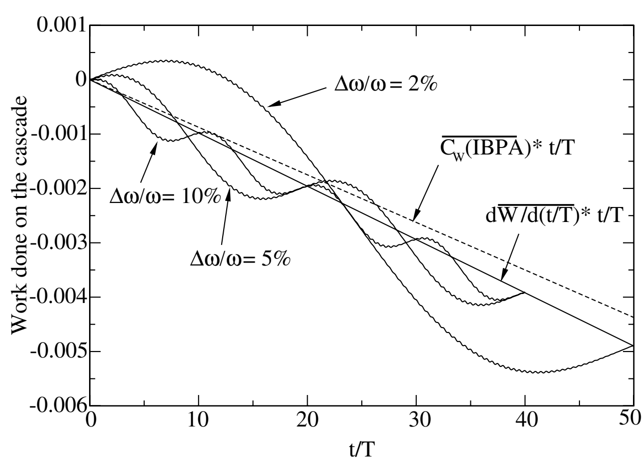

Fig. 2 Computed work on cascade with different amounts of mistuning by uncoupled method.

$$W(t) = \int_0^t C_h \, \mathrm{d}h \tag{4}$$

With uncoupled simulations the solutions is periodic, so that the work performed within a total period, i.e., $W(t + T_{\text{total}}) - W(t)$, is independent of time t where T_{total} is the beating period of the natural frequencies of the neighboring blades [7].

The work coefficient in Eq. (3) can also be interpreted as the average amount of work performed within a period of the tuned system, where the average is performed over the total period.

$$C_W = \frac{T_{\text{tuned}}}{T_{\text{total}}}[W(t + T_{\text{total}}) - W(t)] \tag{5}$$

The results in Fig. 2 were obtained by calculations on four blade passages allowing for the critical IBPA of 270 deg.

The effect of the frequency difference $\Delta\omega$ introduced by mistuning every second blade in the row is apparent in Fig. 2. The phase difference between neighboring blades goes through the whole range between 0 and 360 deg within each period $2\pi/\Delta\omega$. One could think of each blade going through the unstable and stable regions shown in Fig. 1 as the phase difference changes with time. The slope of the curves in Fig. 2 is the instantaneous work W [Eq. (4)] per unit time, which signifies instability when positive and stability when negative. For all cases shown in Fig. 2, the slope averaged over $T_{\text{total}} = 2\pi/\Delta\omega$

$$\overline{\mathrm{d}W/\mathrm{d}t} = \frac{1}{T_{\text{total}}}\int_0^{T_{\text{total}}} \frac{\mathrm{d}W(t)}{\mathrm{d}t}\,\mathrm{d}t = \frac{W(T_{\text{total}})}{T_{\text{total}}} \tag{6}$$

is shown with the straight solid line. For the tuned cascade, from Eq. (5), this average slope is equal to the work coefficient C_W at the given IBPA, divided by the period T_{tuned}.

It is remarkable that within the investigated range of mistuning levels, the work performed on the cascade seems to fall on a common average slope. There is a way to approximate this slope, and therefore predict the average stability of the mistuned cascade, by using only information obtained from the tuned cascade. With alternate mistuning, the IBPA is not constant, but a linear function of time. One might consider the IBPA-averaged work coefficient of the tuned cascade

$$\overline{C_W(\text{IBPA})} = \frac{1}{2\pi}\int_0^{2\pi} C_W(\text{IBPA})\,\mathrm{dIBPA} \tag{7}$$

and compare the average slope $W(T_{\text{total}})/T_{\text{total}}$ shown with the solid line in Fig. 2, with $\overline{C_W(\text{IBPA})}/T_{\text{tuned}}$, shown with the dashed line in Fig. 2. The agreement between the two lines is quite good. The difference between the average slope and the IBPA-averaged work coefficient obtained from Fig. 1 is mainly due to the fact that the phase difference between blade n and blade $n-1$ is given by

$$\begin{aligned}\phi_n - \phi_{n-1} &= \omega t + \phi_{0,n} - (\omega + \Delta\omega)t - \phi_{0,n-1}\\ &= -\Delta\omega t + \Delta\phi_{0,n,n-1}\end{aligned}$$

but between blade $n+1$ and blade n the phase difference becomes

$$\begin{aligned}\phi_{n+1} - \phi_n &= (\omega + \Delta\omega)t + \phi_{0,n+1} - \omega t - \phi_{0,n}\\ &= +\Delta\omega t + \Delta\phi_{0,n+1,n}\end{aligned}$$

where ϕ_0 are initial phase angles. Thus, the phase differences to the suction-side neighbor and to the pressure-side neighbor are different at most times. The influence of the neighbor on the suction side is dominant, but the influence of the pressure-side neighbor and second neighbors explains why the average temporal derivative of the cascade work deviates from the average work coefficient.

With the uncoupled approach the work coefficient in Eq. (3) is used as the single measurement for stability. In the case of frequency mistuning, however, the work coefficient is not a suitable measure because it only provides information about the time-average behavior. Figure 2 shows that even though the overall behavior is

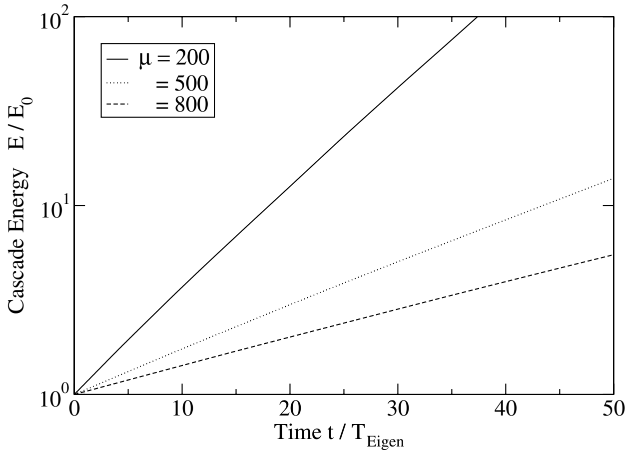

Fig. 3 Total energy of the tuned cascade vs time.

stable, the mistuned cascade exhibits a temporal maximum in work which is higher at lower levels of mistuning. The cascade initially extracts energy from the flow until a maximum is reached before the energy decreases. In an actual machine, the structural motion of the blade is driven by the aerodynamic force. Thus, this maximum energy level corresponds to a maximum oscillation amplitude, which may exceed an allowable level when the mistuning amount is too low. There will be a requirement for a minimum mistuning level at which the cascade can be regarded as stable. A coupled method has to be applied to account for such temporal effects.

Coupled Simulations

The uncoupled method is valid only if the blade mass ratio in Eq. (2) is high enough so that the structural behavior is not affected by aerodynamic forcing. Here, ρ_∞ is the density at the inlet. When investigating the effects of mistuning, the fluid–structure interaction may be significant even at high mass ratios. At a high mass ratio the aerodynamic forcing is small compared to inertial forces. However, if the effect of mistuning is also small, the coupling effect may not be negligible. All following results were obtained by the coupled approach, simultaneously solving for the flow and the structural motion of four blade passages.

Turbine Cascade

In Fig. 3 the total cascade energy is shown over time for the tuned standard configuration 4 and an IBPA of 270 deg. The total energy of the cascade is given by the sum of the kinetic and potential energies of all blades:

$$E = \sum_m \frac{1}{2}\dot{\boldsymbol{q}}_m \cdot \dot{\boldsymbol{q}}_m + \frac{\omega^2}{2}\boldsymbol{q}_m \cdot \boldsymbol{q}_m$$

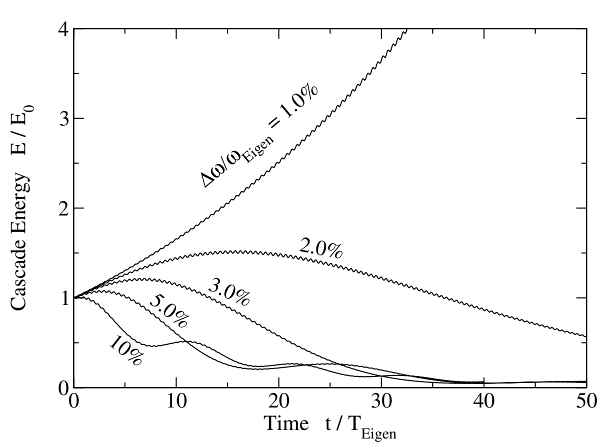

Fig. 4 Time history of the cascade energy for mass ratio 500.

where m is the blade index. The tuned cascade is unstable as was predicted by the uncoupled method. Using the coupled approach we find that the instability increases as the mass ratio is decreased. The cascade energy increases more rapidly at lower mass ratios in Fig. 3, which implies that the fluid–structure interaction has a destabilizing effect.

Figure 4 shows the time history of the total energy of the cascade for a mass ratio of 500 and alternate frequency mistuning by 1, 2, 3, 5, and 10%. For mistuning amounts above 2% the cascade is stable. However, there is a maximum total energy, i.e., a maximum deflection, which increases as $\Delta\omega$ is decreased. Even for high mass ratios, insufficient mistuning will bring the system to being unstable, thus over a long time period, the energy levels differ exponentially. This confirms the conjecture drawn from uncoupled results as discussed in the preceding section and also in [7].

However, Fig. 4 also shows that mistuning by only 1% does not stabilize the cascade even over long periods of time. The reason for this surprising result is given by the influence of fluid–structure coupling on the blade oscillation frequency. It should be noted that due to fluid–structure interaction a blade is not oscillating at its structural eigenfrequency. The actual oscillation frequency differs from the eigenfrequency, and this difference increases as the mass ratio is decreased. With this test case, the influence of fluid–structure coupling apparently decreases the effective mistuning. Because this phenomenon is due to fluid–structure interaction, it cannot be predicted by an uncoupled approach.

More detailed plots of the time history of the cascade energy are given in Figs. 5–7 for the mass ratios of 200, 500, and 800. Note that the energy is plotted on a logarithmic scale, where a linear behavior indicates exponential decay or growth. For each mass ratio, the

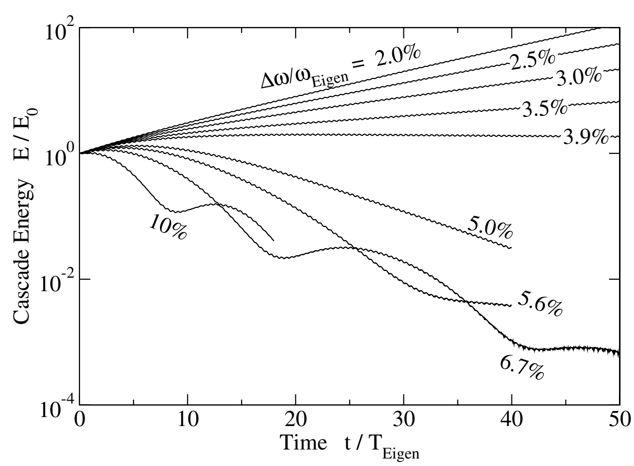

Fig. 5 Total energy of the cascade vs time for mass ratio 200.

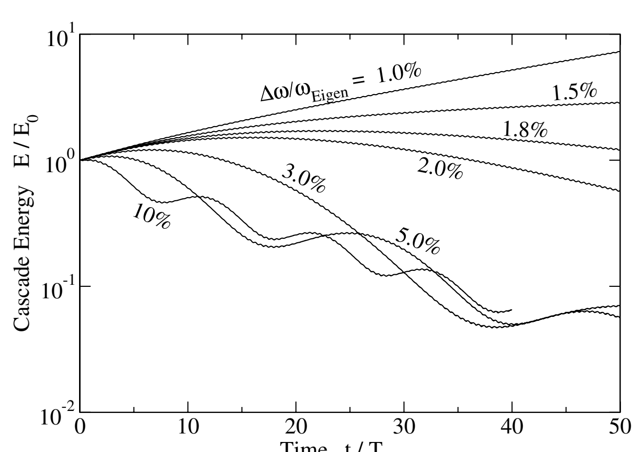

Fig. 6 Total energy of the cascade vs time for mass ratio 500.

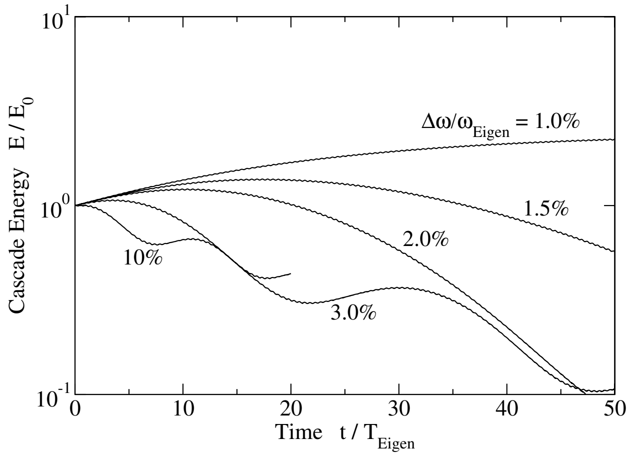

Fig. 7 Total energy of the cascade vs time for mass ratio 800.

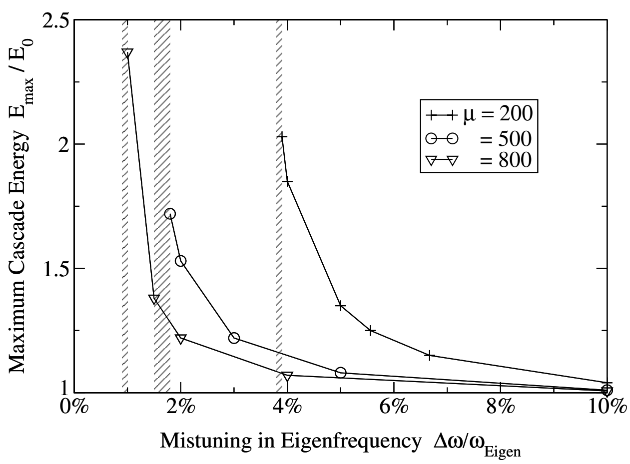

Fig. 8 Maximum cascade energy over mistuning amount.

energy is plotted for several mistuning amounts, and an attempt was made to find the minimum amount necessary for stabilization. We can see from Figs. 5–7 that this minimum decreases with increasing mass ratio. This is because the coupling between fluid and structure decreases as the mass ratio is increased. In the limit of an infinite mass ratio, the aerodynamic equations and the structural equations are decoupled and we would expect to find the result of the uncoupled method, i.e., stabilization at all mistuning amounts.

A temporal maximum of the cascade energy is present at all stable solutions and is plotted over the amount of mistuning in Fig. 8 for the three mass ratios μ. Again, this figure shows that the cascade is unstable below a certain finite amount of mistuning, which becomes smaller as the mass ratio is increased. Stable solutions exhibit a maximum energy, which increases and approaches infinity as the mistuning amount is decreased to a lower limit. Rather than using the damping coefficient or the mistuning amount, it is more appropriate to define the stability limit as a horizontal line in Fig. 8 to mark a maximum allowable cascade energy. To yield accurate quantitative solutions, however, a three-dimensional configuration should be investigated solving the Navier–Stokes equations coupled to the structural equations of three-dimensional blades.

In a real cascade, the actual flow-on oscillation frequencies are not known a priori and are hard to control in advance. Structural eigenfrequencies can be controlled more easily by controlling the blade stiffness through the choice of blade material and structure. For this reason, by *mistuning* we mean mistuning in structural eigenfrequency, not in actual oscillation frequency. Because of fluid–structure interaction, the actual oscillation frequency is not equal to the structural eigenfrequency.

Figure 9 helps to explain the existence of the lower limit for the mistuning amount. The effective mistuning, i.e., the difference

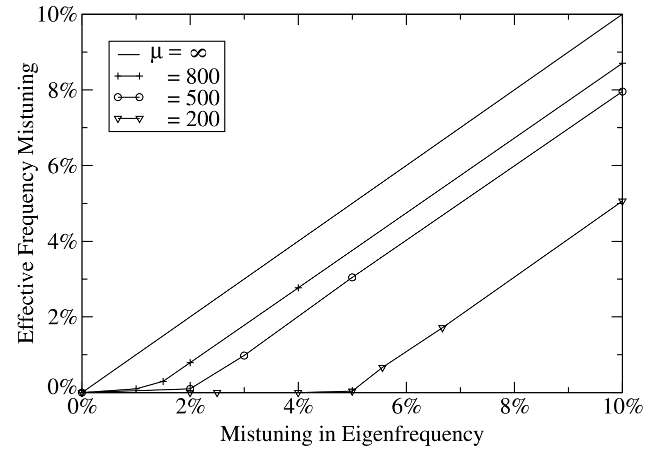

Fig. 9 Effective mistuning vs nominal mistuning for mass ratios 200 and 500.

present in the actual oscillation frequencies of adjacent blades, is plotted over the nominal mistuning, i.e., the applied difference of structural eigenfrequencies. The actual frequencies used to determine the effective frequency mistuning in Fig. 9 are determined by Fourier transforms of the computed time histories. They were found to be amplitude and damping independent. The results for mass ratios of 200, 500, and 800 are shown together with the uncoupled solution ($\mu = \infty$). For the uncoupled case, where the blades are indeed oscillating at their eigenfrequencies, the applied amount of mistuning is identical to the mistuning of the actual oscillation frequency.

Note that for finite mass ratios, below a certain structural mistuning amount, the cascade stays essentially tuned and is therefore unstable. Interestingly, as the structural mistuning is increased above this limit, the actual mistuning resumes to increase at the originally expected rate. The influence of fluid–structure coupling appears to add a "free-play" of mistuning to the system. This behavior can be understood in analogy to the well-known phenomenon of frequency lock-in as it appears in vortex-induced vibrations [16]. In vortex-induced vibrations, a structure is observed to synchronize with the flow oscillation if the structural eigenfrequency is close enough to the eigenfrequency of the flow. In the case of cascade flutter described here, the flow does not exhibit oscillatory behavior such as vortex shedding. However, it provides aerodynamic coupling between multiple blades. When the structural differences between adjacent blades are small enough to be suppressed by this aerodynamic coupling, all blades lock in to the same oscillation frequency. This lock-in frequency may be regarded as an eigenfrequency of the complete aeroelastic system, and may not be inherent to either the flow or the structure individually. Thus it can only be predicted by a coupled aeroelastic solution.

Compressor Cascade

As a second test case the tenth standard configuration by Fransson and Verdon [17] was chosen, which is a cascade of modified NACA 0006 profiles. At an inlet Mach number of 0.5, an inlet angle of 48 deg, and torsional motion around midchord, this cascade exhibits flutter at interblade phase angles around 90 deg. The following results were obtained by calculations on four blade passages with the coupled approach.

As with the turbine cascade presented in the preceding section, the stability of the tuned compressor cascade decreases as the mass ratio is decreased in Fig. 10. The total energy of the tuned cascade increases more rapidly with stronger fluid–structure coupling.

Figures 11 and 12 show the time histories of the cascade energy for the two mass ratios of 180 and 350 and for various amounts of alternate mistuning. Relatively large amounts of mistuning are necessary to stabilize this compressor cascade, compared to the turbine cascade. Because compressor blades have a lower mass ratio than solid turbine blades, the effects of fluid–structure interaction are

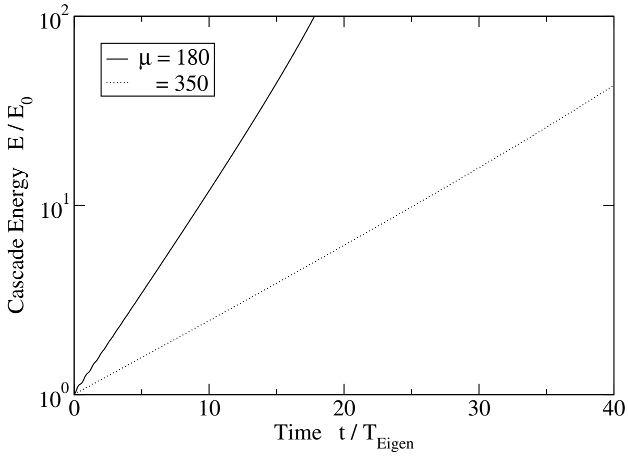

Fig. 10 Total energy of the tuned cascade vs time.

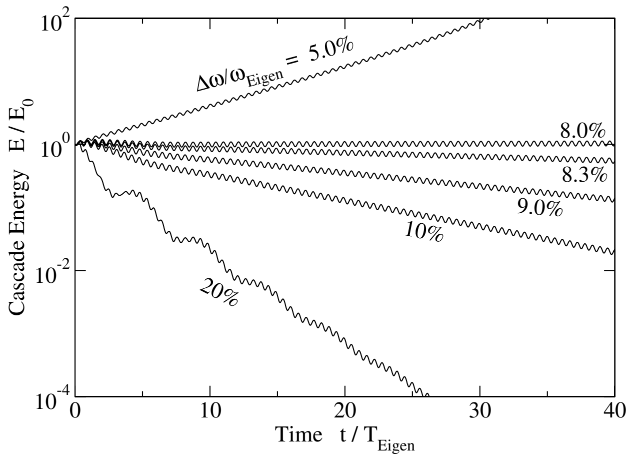

Fig. 11 Total energy of the cascade vs time for mass ratio 180.

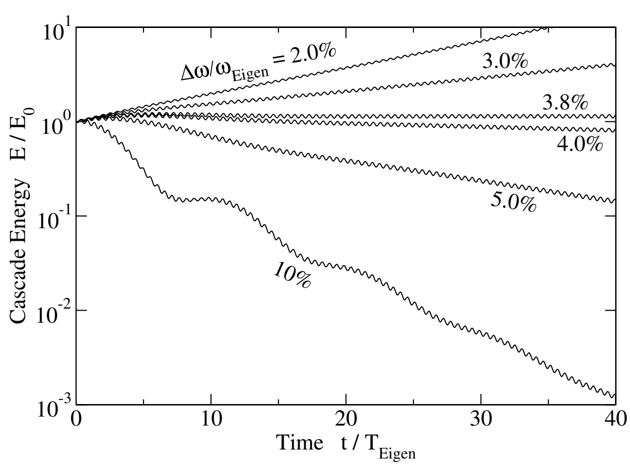

Fig. 12 Total energy of the cascade vs time for mass ratio 350.

stronger. Coupling reduces the effective mistuning. Therefore, the stronger the coupling is, the more mistuning in eigenfrequency is needed to overcome this effect. Also the compressor cascade exhibits a maximum cascade energy, which decreases as the amount of mistuning is increased in Fig. 13.

Qualitatively, the behavior of the compressor cascade is very similar to the behavior of the turbine test case. Because of the lower mass ratio of the compressor blades, however, there is stronger fluid–structure interaction than with the turbine cascade. Therefore, the minimum amount of mistuning required to stabilize the compressor cascade is higher than that for the turbine cascade.

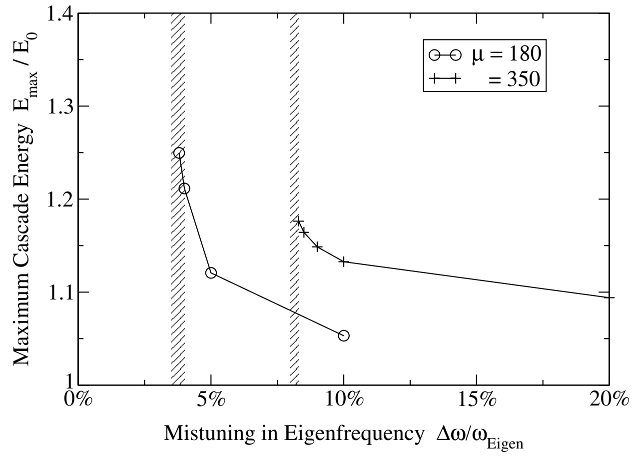

Fig. 13 Maximum cascade energy over mistuning amount.

Conclusions

As found in a previous study [7] and confirmed with the present work, frequency mistuning may stabilize a cascade by causing temporally changing phase differences between adjacent blades. These temporal changes have an averaging effect on the stability of unstable and stable oscillation modes over the complete IBPA range. With alternate mistuning, the resulting work coefficient, however, is not exactly given by the IBPA-averaged work coefficient of the tuned cascade. This discrepancy is due to different contributions of the suction-side neighbor and the pressure-side neighbor of each blade.

In the present work, results by the coupled method and alternate frequency mistuning confirm the existence of a maximum total energy associated with a temporal maximum in blade deflection, even for stable situations predicted by the traditional uncoupled energy method. Thus, depending on the mass ratio, a minimum mistuning level $\Delta\omega$ is required, corresponding to a maximum allowable deflection amplitude. This maximum amplitude decreases as the amount of mistuning is increased.

Furthermore, computations reveal that fluid–structure coupling reduces the actual amount of mistuning in a turbine or a compressor system. When fluid–structure coupling is present, the effective amount of mistuning, as determined by the difference in the actual oscillation frequencies of the neighboring blades, is found to be smaller than the nominal mistuning amount, defined as the difference of the structural eigenfrequencies of the blades. Analogous to the lock-in phenomenon in vortex-induced vibration, different dynamic systems (i.e., blades) with slightly different eigenfrequencies are locked in to the same oscillation frequency. Here, fluid–structure coupling acts to suppress minor structural differences between adjacent blades. By this mechanism a structurally mistuned cascade is able to oscillate in an effectively tuned fashion. The minimum mistuning amount, below which this frequency lock-in happens, increases with decreasing mass ratio of the blades because of an increased influence of fluid–structure coupling. This threshold of minimum mistuning amount can only be predicted by using a coupled fluid–structure simulation method because it is a phenomenon of the coupled fluid–structure system.

References

[1] Whitehead, D. S., "Effect of Mistuning on Vibration of Turbo-Machine Blades Induced by Wakes," *Journal of Mechanical Engineering Science*, Vol. 8, No. 1, 1966, pp. 15–21.

[2] Ewins, D., "The Effect of Detuning upon the Forced Vibration of Bladed Disks," *Journal of Sound and Vibration*, Vol. 9, No. 1, 1969, pp. 65–79.

[3] Kaza, K. R. V., and Kielb, R. E., "Flutter of Turbofan Rotors with Mistuned Blades," *AIAA Journal*, Vol. 22, No. 11, 1984, pp. 1618–1625.

[4] Imregun, M., and Ewins, D. J., "Aeroelastic Vibration Analysis of Tuned and Mistuned Blade Systems," *Unsteady Aerodynamics of*

Turbomachines and Propellers, Symposium Proceedings, Cambridge Univ., Cambridge, England, U.K., Sept. 1984, pp. 149–161.
[5] Mehmed, O., and Murthy, D. V., "Experimental Investigation of Propfan Aeroelastic Response in Off-Axis Flow with Mistuning," *Journal of Propulsion and Power*, Vol. 7, No. 1, 1991, pp. 90–98.
[6] Nowinski, M., and Panovsky, J., "Flutter Mechanisms in Low Pressure Turbine Blades," *Journal of Engineering for Gas Turbines and Power*, Vol. 122, No. 1, 2000, pp. 82–88.
[7] Sadeghi, M., and Liu, F., "A Method for Calculating Mistuning Effects on Cascade Flutter," *AIAA Journal*, Vol. 39, No. 1, 2001, pp. 22–28.
[8] Bölcs, A., and Fransson, T. H., "Aeroelasticity in Turbomachines—Comparison of Theoretical and Experimental Cascade Results," *Communication du Laboratoire de Thermique Appliquée et de Turbomachines*, No. 13, Ecole Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland, 1986.
[9] Campobasso, M. S., and Giles, M. B., "Flutter and Forced Response of Mistuned Turomachinery," *Proceedings of the 9th ISUAAAT*, Presses Universitaires de Grenoble, Grenoble, Sept. 2000, pp. 885–896.
[10] Sadeghi, M., and Liu, F., "Investigation of Non-Linear Flutter by a Coupled Aerodynamics and Structural Dynamics Method," AIAA Paper 2001-0573, 2001.
[11] Sadeghi, M., and Liu, F., "Computation of Cascade Flutter by Uncoupled and Coupled Methods," *International Journal of Computational Fluid Dynamics*, Vol. 19, No. 8, Nov. 2005, pp. 559–569.
[12] Wilcox, D. C., "Reassessment of the Scale-Determining Equation for Advanced Turbulence Models," *AIAA Journal*, Vol. 26, No. 11, 1988, pp. 1299–1310.
[13] Ji, S., and Liu, F., "Flutter Computation of Turbomachinery Cascades Using a Parallel Unsteady Navier-Stokes Code," *AIAA Journal*, Vol. 37, No. 3, 1999, pp. 320–327.
[14] Jameson, A., Schmidt, W., and Turkel, E., "Numerical Solutions of the Euler Equations by Finite Volume Methods Using Runge-Kutta Time-Stepping Schemes," AIAA Paper 81-1259, 1981.
[15] Jameson, A., "Time Dependent Calculations Using Multigrid, with Applications to Unsteady Flows Past Airfoils and Wings," AIAA Paper 91-1596, 1991.
[16] Williamson, C. H. K., and Govardhan, R., "Vortex-Induced Vibrations," *Annual Review of Fluid Mechanics*, Vol. 36, 2004, pp. 413–455.
[17] Fransson, T. H., and Verdon, J. M., "Updated Report on Standard Configurations for Unsteady Flow Through Vibrating Axial-Flow Turbomachine Cascades," Royal Institute of Technology Report, Stockholm, Sweden, 1991.

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